MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

BTS3014 –TIME SERIES ANALYSIS AND FORECASTING

(All sections / Groups)

8 MARCH 2018
9.00 a.m. - 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **FOURTEEN** pages with **TWO** sections only.
   Section A – Twenty (25) Multiple Choice Questions
   Section B – Three (3) Structured Questions

2. Attempt **ALL** questions in both **Sections A and B**. The distribution of the marks for each question is given.

3. Answer **Section A** in the MCQ answer sheet provided and **Section B** in the answer booklet provided.

4. Statistical tables and formulas are attached.

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SECTION A: MULTIPLE CHOICE QUESTIONS (25 MARKS)

1. What is most likely to be the major difference between forecasting sales of a private business versus forecasting the demand of a public good supplied by a governmental agency?
   A) Amount of data available.
   B) Underlying economic relationships.
   C) Lack of market-determined price data for public goods.
   D) Last of historical data.

2. For the forecasting process to be effective, internal data should be
   A) kept on an annual basis.
   B) highly aggregated.
   C) sampled once a year.
   D) highly disaggregated.

3. You are given a time series of sales data with 10 observations. You construct forecasts according to last period's actual level of sales plus the most recent observed change in sales. How many data points will be lost in the forecast process relative to the original data series?
   A) One.
   B) Two.
   C) Three.
   D) Zero.

4. Which of the following measures of forecast accuracy can be used to compare "goodness of fit" across different sized random variables?
   A) Mean Absolute Error.
   B) Mean Absolute Percentage Error.
   C) Mean Squared Error.
   D) Root Mean Squared Error.

5. Which measure of forecast accuracy is analogous to standard deviation?
   A) Mean Absolute Error.
   B) Mean Absolute Percentage Error.
   C) Mean Squared Error.
   D) Root Mean Squared Error.

6. Forecasting January sales based on the previous month's level of sales is likely to lead to error if the data are _____.
   A) Stationary.
   B) Non-cyclical.
   C) Seasonal.
   D) Irregular.

Continued...

7. A large sample of *X*-*Y* data values are analysed and reveal a correlation coefficient of -0.88. Which statement is correct?
   A) If r had been +0.88, the correlation would have been much stronger.
   B) The correlation is weak because r is less than -1.
   C) A fairly strong negative linear relationship exists.
   D) A weak negative relationship exists.

8. Autocorrelation refers to the correlation between a variable and:
   A) itself.
   B) another very similar variable.
   C) itself when lagged one or more periods.
   D) another variable when the analysis is done on a computer.

9. As an example of how Holt-Winter's smoothing model deals with seasonal data, how would actual quarter four sales of a retail firm be deseasonalised?
   A) It would be divided by a seasonal factor.
   B) It would be multiplied by a seasonal factor.
   C) It would be added with a seasonal factor.
   D) It would be subtracted from a seasonal factor.

10. What is a common way forecasters attempt to eliminate heteroscedasticity?
    A) Transform the data by first differencing.
    B) Deseasonalise the data.
    C) Transform the data using logarithms.
    D) Apply Ordinary Least Squares.

11. Cross-sectional regression models linking personal disposable income to consumption expenditure are likely to be hampered by
    A) nonlinearity.
    B) homoscedasticity.
    C) autocorrelation.
    D) heteroscedasticity.

12. How many critical regions does the Durbin-Watson test have?
    A) two.
    B) three.
    C) four.
    D) five.

13. If R^2 is 0.95 in a simple regression model, it can be said that:
    A) X and Y have a correlation of 0.95.
    B) the relationship between X and Y is positive.
    C) 5 percent of Y's variability is caused by variability in X.
    D) 95% of Y's variability can be explained by X's variability.

Continued...

14. When autocorrelation is present in time series data, which underlying assumption is violated?
    A) The population of Y values is normally distributed about the population regression line.
    B) The dispersion of population data points around the population regression line remains constant everywhere along the line.
    C) The error terms are independent of each other.
    D) A linear relationship exists between X and Y in the population.

15. Use the Akaike Information Criterion
    A) in observational studies when there are large numbers of variables.
    B) in exploratory studies when you have no a priori hypotheses.
    C) in experimental studies when you are testing few effects.
    D) to select the correct degrees of freedom to use in evaluating summary statistics.

16. Large and complicated forecasting models
    A) are expensive to maintain.
    B) are hard to explain to upper-level management.
    C) tend to put too much emphasis on quantitative aspects of forecasting.
    D) All of the above.

17. In a regression of sales on income and seasonal dummy variables for a quarterly time series, a negative sign of the quarter 3 dummy variable means
    A) sales for quarter three are negative.
    B) sales for quarter three are below average.
    C) sales for quarter three are average below that of the base quarter.
    D) sales for quarter three are above average.

18. The inclusion of seasonal dummy variables to a multiple regression model may help eliminate
    A) autocorrelation if the data are characterised by seasonal fluctuations.
    B) perfect multicollinearity.
    C) near multicollinearity.
    D) bias in OLS slope estimates caused by autocorrelation.

19. Over a long period of time, if measured correctly, cycle factors should average
    A) zero.
    B) one.
    C) two.
    D) four.

Continued...

20. A researcher mistakenly uses deseasonalised data in calculating the seasonal factors. If she found apparent seasonal behaviour, this is best attributed to
    A) the business cycle.
    B) trend.
    C) random noise.
    D) seasonality.

21. The following specific seasonal factors were estimated for the month of October:

    65.4 76.8 66.9 72.6 70.0

    If the adjustment is 0.98 and the modified mean is used, and if the expected trend for October is RM800, what is the seasonally adjusted forecast?
    A) RM 570.00.
    B) RM 561.00.
    C) RM 551.20.
    D) RM 1,168.8.

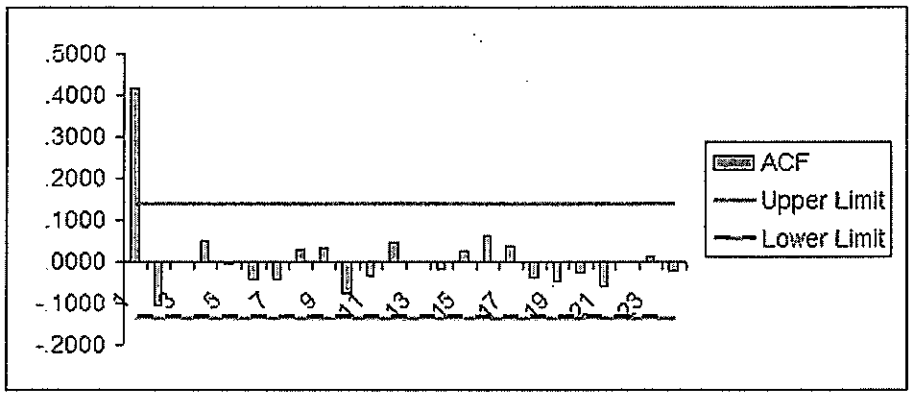
22. Besides using sophisticated ARIMA-type models capable of internally handling data seasonality, an alternative is to use
    A) seasonal dummy variables.
    B) trend dummy variables.
    C) deseasonalised data, and then reseasonalise to generate forecasts.
    D) Holt's smoothing.

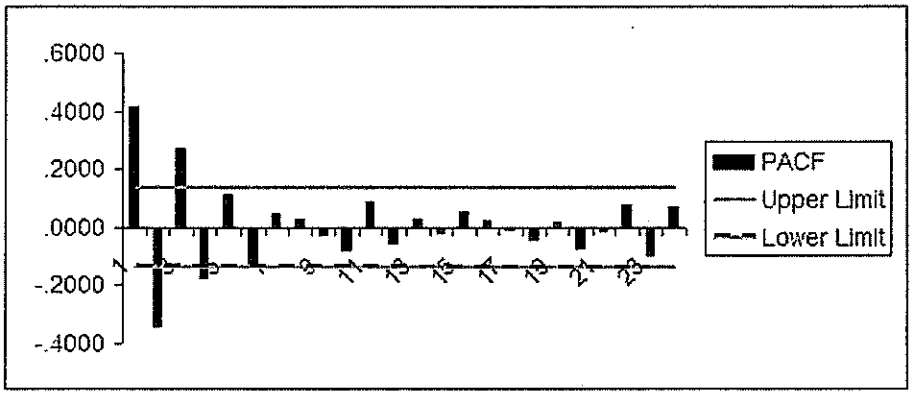
23. Which of the following best describes the autocorrelation function (ACF) of a nonstationary time series?
    A) The ACF has coefficients that very gradually go to zero.
    B) The ACF has a spurious pattern of spikes as lags increase.
    C) The null of zero autocorrelation is rejected for a significant amount of lags.
    D) All of the above.

24. Which of the following models does **NOT** require stationary data?
    A) Simple Exponential Smoothing.
    B) Time Series Decomposition.
    C) Linear Trend Regression.
    D) Moving Averages.

Continued...

25.





The plot above

A) represents a classic AR(1) model configuration.
B) represents a classic MA(1) model configuration.
C) represents a classic AR(4) model configuration.
D) Represents a classic MA(4) model configuration.

Continued...

SECTION B: STRUCTURED QUESTIONS (75 MARKS)

Question One (25 marks)

(a) Discuss how you would identify a moving-average process for a given time series. In particular, what techniques are available for selecting the correct order of the process? (9 marks)

(b) A time series model has been fit and checked with historical data yielding $Y_t = 50 + 0.45Y_{t-1} + \varepsilon_t$

(i) Suppose at time t = 60, the observation is $Y_{60} = 57$. Determine forecast for periods 61, 62, and 63 from origin 60. (6 marks)

(ii) Suppose the observed value of $Y_{61} = 59$. Update the forecasts for periods 62 and 63. (4 marks)

(c) Write the expressions for the following models:
(i) MA(3) (2 marks)
(ii) ARMA(1,2) (2 marks)
(iii) ARIMA(2,1,2) (2 marks)

Question Two (25 marks)

(a) The following quantity supplied of beef and price of beef have been observed in the XYZ Company over 11 years:

Year	Quantity supplied	Price
2007	4	6
2008	6	10
2009	10	10
2010	8	8
2011	6	8
2012	10	12
2013	14	10
2014	12	10
2015	14	12
2016	16	12
2017		14

(i) Find the forecast values of the quantity supplied from 2007 to 2016 by using Naïve trend model. (2 marks)

Continued...

(ii) By regressing quantity supplied on price, you obtain the following regression result:
$\widehat{Quantity\ Supplied}_t = -5.4157 + 1.5730 * Price_t$
Find the forecast values of the quantity supplied from 2007 to 2016 by using the regression result obtained. (3 marks)

(iii) Evaluate these forecasting methods (from part (i) to (ii)) using Mean Absolute Error (MAE) and Mean Square Error (MSE). Show all calculations. (8 marks)

(iv) Forecast the quantity supplied for year 2017 using the best technique (from part (iii)). (2 marks)

(b) Seasonal factors of Quarters 2 and 4 for sales of BB baby foods were reported to be:

Year	Seasonal Factor, Q2	Seasonal Factor, Q4
2006	1.25	0.98
2007	1.13	0.87
2008	1.03	0.60
2009	1.32	1.23
2010	1.24	0.62
2011	0.94	0.49
2012	1.88	0.88
2013	1.02	1.20
2014	1.41	1.14
2015	1.79	0.97

(i) What is the unadjusted seasonal index for Q2 and Q4 sales? (2 marks)

(ii) If the sum of seasonal index numbers is 3.98, find the adjusted Q2 and Q4 seasonal index. (4 marks)

(iii) Using the multiplicative model, provide forecasts for quarters 2 and 4 of 2016 if the fitted trend line is 80.1736 + 5.5959**Trend*. [Hint: the cyclical index and irregular component are both set to the value of 1.0.] (4 marks)

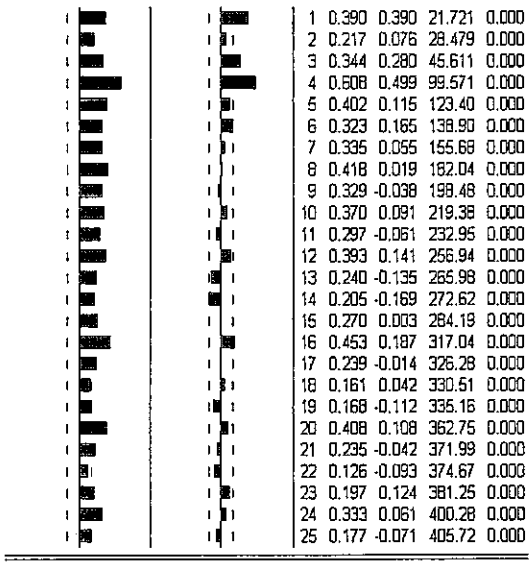
Continued...

Question Three (25 marks)

(a) The Sample Correlograms (up to 25 lags) for the time series Money Supply (*M2*) and Gross Domestic Product (*GDP*) are shown as below:

Time series: *M2*

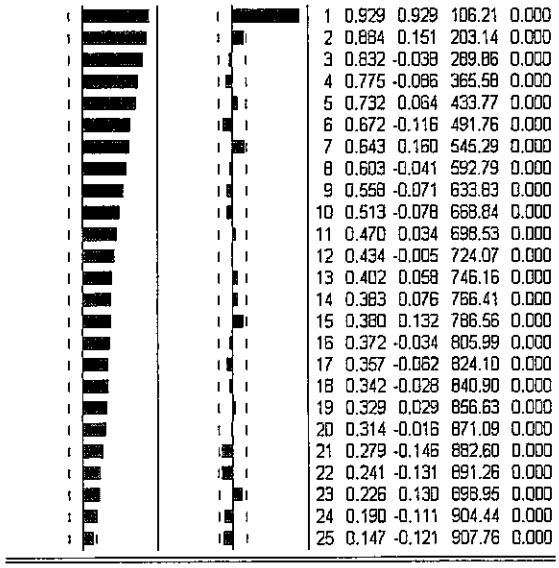
Sample: 1971:1 2005:4
Included observations: 140



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.390	0.390	21.721	0.000
		2	0.217	0.076	28.479	0.000
		3	0.344	0.280	45.611	0.000
		4	0.508	0.499	99.571	0.000
		5	0.402	0.115	123.40	0.000
		6	0.323	0.165	138.90	0.000
		7	0.335	0.055	155.68	0.000
		8	0.418	0.019	182.04	0.000
		9	0.329	-0.038	198.48	0.000
		10	0.370	0.091	219.38	0.000
		11	0.297	-0.061	232.95	0.000
		12	0.393	0.141	256.94	0.000
		13	0.240	-0.135	265.98	0.000
		14	0.205	-0.169	272.62	0.000
		15	0.270	0.003	284.18	0.000
		16	0.453	0.187	317.04	0.000
		17	0.239	-0.014	326.28	0.000
		18	0.161	0.042	330.51	0.000
		19	0.168	-0.112	335.16	0.000
		20	0.408	0.108	362.75	0.000
		21	0.235	-0.042	371.99	0.000
		22	0.126	-0.093	374.67	0.000
		23	0.197	0.124	381.25	0.000
		24	0.333	0.061	400.28	0.000
		25	0.177	-0.071	405.72	0.000

Time series: *GDP*

Sample: 1971:1 2005:4
Included observations: 120



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.929	0.929	106.21	0.000
		2	0.884	0.151	203.14	0.000
		3	0.832	-0.038	289.86	0.000
		4	0.775	-0.086	365.58	0.000
		5	0.732	0.064	433.77	0.000
		6	0.672	-0.116	491.76	0.000
		7	0.643	0.160	545.29	0.000
		8	0.603	-0.041	592.79	0.000
		9	0.558	-0.071	633.63	0.000
		10	0.513	-0.078	668.84	0.000
		11	0.470	0.034	698.53	0.000
		12	0.434	-0.005	724.07	0.000
		13	0.402	0.058	746.16	0.000
		14	0.383	0.076	766.41	0.000
		15	0.360	0.132	786.56	0.000
		16	0.372	-0.034	805.99	0.000
		17	0.357	-0.062	824.10	0.000
		18	0.342	-0.028	840.90	0.000
		19	0.329	0.029	856.63	0.000
		20	0.314	-0.016	871.09	0.000
		21	0.279	-0.146	882.60	0.000
		22	0.241	-0.131	891.26	0.000
		23	0.226	0.130	898.96	0.000
		24	0.190	-0.111	904.44	0.000
		25	0.147	-0.121	907.76	0.000

What general pattern do you see? Intuitively, which one(s) of these time series seem to be stationary? Why? Explain your answer statistically. (5 marks)

(b) The Augmented Dickey-Fuller (ADF) test results for the two time-series (*M2* and *GDP*) are as below:

$$\Delta M2_t = 312.55 - 0.20M2_{t-1} - 0.63\Delta M2_{t-1} - 0.64\Delta M2_{t-2} \qquad R^2 = 0.53$$
$$\tau \qquad (1.55) \quad (-2.13) \quad (-6.11) \qquad (-7.17)$$

$$\Delta^2 M2_t = 35.58 - 3.11\Delta M2_{t-1} + 1.33\Delta^2 M2_{t-1} - 0.59\Delta^2 M2_{t-2} \qquad R^2 = 0.81$$
$$\tau \qquad (0.23) \quad (-16.86) \quad (10.13) \qquad (-7.85)$$

$$\Delta GDP_t = -734.04 - 0.03GDP_{t-1} - 0.15\Delta GDP_{t-1} + 14.57Trend \qquad R^2 = 0.06$$
$$\tau \qquad (-1.35) \quad (-0.97) \quad (-1.62) \qquad (1.89)$$

$$\Delta^2 GDP_t = -482.13 - 1.09\Delta GDP_{t-1} - 0.07\Delta^2 GDP_{t-1} + 9.05Trend \qquad R^2 = 0.60$$
$$\tau \qquad (-0.99) \quad (-7.40) \quad (-0.78) \qquad (1.62)$$

Continued...

where *M2* = money supply, *GDP* = gross domestic product, Δ = first-difference operator and Δ^2 = second-difference operator.

The critical τ values (at 10%) are –2.57 and –3.16 for models without trend and with trend, respectively.

From the unit root tests,

(i) Is *M2* integrated of order zero I(0), order one I(1) or order two I(2)? Explain. (3 marks)

(ii) Is *GDP* integrated of order zero I(0), order one I(1) or order two I(2)? Explain. (3 marks)

(c) Company XYZ's management has long been interested in more accurately predicting the level of business obtained from its customers in the attempt to provide a better basis for production controls and marketing efforts. Researchers at Company XYZ proposed that a multiple regression analysis should be attempted to predict the product usage levels of the customers based on their perceptions lf Company XYZ's performance. In addition, the researchers were also interested in identifying the factors that led to increased product usage for application in differentiated marketing campaigns. Researchers selected product usage level as the dependent variable to be predicted by independent variables representing perceptions of Company XYZ's performance. Seven variables were included as independent variables, namely *Delivery Speed*, *Price Level*, *Price Flexibility*, *Manufacturer Image*, *Service*, *Salesforce Image* and *Product Quality*. Results from the analysis are listed below:

Output 1: Correlations

	Usage Level	Delivery Speed	Price Level	Price Flexibility	Manufacturer Image	Service	Salesforce Image	Product Quality
Usage Level	1.000							
Delivery Speed	0.676	1.000						
Price Level	0.082	-0.349	1.000					
Price Flexibility	0.559	0.509	-0.487	1.000				
Manufacturer Image	0.224	0.05	0.272	-0.116	1.000			
Service	0.701	0.612	0.513	0.067	0.299	1.000		
Salesforce Image	0.256	0.077	0.186	-0.034	0.788	0.241	1.000	
Product Quality	-0.192	-0.483	0.470	-0.448	0.200	-0.055	0.177	1.000

Continued...

Output 2: Model Summary[d]

Model	R	R^2	$\widehat{R}^2$
1	0.701[a]	0.491	0.486
2	0.869[b]	0.755	0.750
3	0.877[c]	0.768	0.761

Note:
[a]. Predictors: (Constant), Service
[b]. Predictors: (Constant), Service, Price Flexibility
[c]. Predictors: (Constant), Service, Price Flexibility, Salesforce Image
[d]. Dependent Variable: Usage Level

Output 3: ANOVA[d]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	3927.309	1	3927.309	94.525	0.000[a]
	Residual	4071.691	98	41.548		
	Total	7999.000	99			
2	Regression	6036.513	2	3018.256	149.184	0.000[b]
	Residual	1621.487	97	20.232		
	Total	7999.000	99			
3	Regression	6145.700	3	2048.567	106.115	0.000[c]
	Residual	1853.300	96	19.305		
	Total	7999.000	99			

Note:
[a]. Predictors: (Constant), Service
[b]. Predictors: (Constant), Service, Price Flexibility
[c]. Predictors: (Constant), Service, Price Flexibility, Salesforce Image
[d]. Dependent Variable: Usage Level

Output 4: Coefficients[a]

Model		Unstandardised Coefficients		Standardised Coefficients		t	Sig.	Collinearity Statistics	
		B	Std. Error	B	Std. Error			Tolerance	VIF
1	(Constant)	21.653	2.596			8.341	0.000		
	Service	8.384	0.862	0.701	0.072	9.722	0.000	1.000	1.000
2	(Constant)	-3.489	3.057			-1.141	0.257		
	Service	7.974	0.603	0.666	0.050	13.221	0.000	0.996	1.004
	Price Flexibility	3.336	0.327	0.515	0.050	10.210	0.000	0.996	1.004
3	(Constant)	-6.520	3.247			-2.008	0.047		
	Service	7.621	0.607	0.637	0.051	12.547	0.000	0.936	1.068
	Price Flexibility	3.376	0.320	0.521	0.049	10.562	0.000	0.993	1.007
	Salesforce Image	1.406	0.591	0.121	0.051	2.378	0.019	0.939	1.064

Note:
[a]. Dependent Variable: Usage Level

Continued...

Based on **Outputs 1 to 4**,

(i) Interpret the regression model and assess the overall model fit by referring to the coefficient of determination, adjusted coefficient of determination, and statistical significance of regression coefficients. (6 marks)

(ii) Evaluate the relative importance of the independent variables with the unstandardised beta coefficients. (3 marks)

(iii) Assume that a customer rated Company XYZ with a value of 4.0 for each of the three independent variables shown in Model 3 of **Output 4**. Predict the product usage level for that particular customer. (3 marks)

(iv) Assess the degree of multicollinearity and determine its effects. (2 marks)

End of Page

Statistical Tables

Appendix 1: t-Table

two tails	0.2	0.1	0.05	0.02	0.01
One tail	0.1	0.05	0.025	0.01	0.005
df					
10	1.37	1.81	2.23	2.76	3.17
20	1.33	1.72	2.09	2.53	2.84
30	1.31	1.70	2.04	2.46	2.75
40	1.30	1.68	2.02	2.42	2.70
50	1.30	1.68	2.01	2.40	2.68
60	1.30	1.67	2.00	2.39	2.66
75	1.29	1.67	1.99	2.38	2.64
100	1.29	1.66	1.98	2.36	2.63
120	1.29	1.66	1.98	2.36	2.62
200	1.29	1.66	1.98	2.36	2.62

Appendix 2: Durbin Watson d Table

Critical Values for the Durbin-Watson Statistic (d)

Level of Significance $\alpha = 0.05$

n	*k* = 1		*k* = 2		*k* = 3		*k* = 4		*k* = 5	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
10	0.88	1.32	0.70	1.64	0.53	2.02	0.38	2.41	0.24	2.82
15	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
20	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
25	1.29	1.41	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78
150	1.72	1.75	1.71	1.76	1.69	1.77	1.68	1.79	1.66	1.80
200	1.76	1.78	1.75	1.79	1.74	1.80	1.73	1.81	1.72	1.82

where n = number of observations and k = number of independent variables, excluding the intercept.

Formulas

Error, $\varepsilon_t = y_t - \hat{y}_t$

Mean Absolute Error, $MAE = \frac{1}{n}\sum_{t=1}^{n}|\varepsilon_t|$

Mean Squared Error, $MSE = \frac{1}{n}\sum_{t=1}^{n}\varepsilon_t^2$

Root Mean Squared Error, $RMSE = \sqrt{MSE}$

Mean Absolute Percentage Error,

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}|PE_t| = \frac{1}{n}\sum_{t=1}^{n}\left|\left(\frac{\varepsilon_t}{y_t}\right)x100\right|$$

Mean Percentage Error,

$$MPE = \frac{1}{n}\sum_{t=1}^{n}PE_t = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{\varepsilon_t}{y_t}\right)x100$$

Theil's U-statistics,

$$U = \sqrt{\frac{\sum_{t=1}^{n}\varepsilon_t^2}{\sum_{t=1}^{n}(y_t - y_{t-1})^2}}$$

Naïve model, $\hat{Y}_{t+1} = Y_t$

Naïve trend model,

$\hat{Y}_{t+1} = Y_t + (Y_t - Y_{t-1})$

Naïve rate of change model,

$$\hat{Y}_{t+1} = Y_t * \frac{Y_t}{Y_{t-1}}$$

Naïve seasonal model for quarterly data,

$\hat{Y}_{t+1} = Y_{t-3}$

Naïve trend and seasonal model for quarterly data,

$$\hat{Y}_{t+1} = Yt-3+\frac{(Yt-Yt-4)}{4}$$

Simple average model, $\hat{Y}_{t+1} = \frac{1}{t}\sum_{i=1}^{t}Y_t$

Moving average model for k time periods,

$$\hat{Y}_{t+1} = \frac{Yt + Yt-1 + ... + Yt-k+1}{k}$$

Double moving average,

$$M_t = \hat{Y}_{t+1} = \frac{Yt + Yt-1 + ... + Yt-k+1}{k}$$

$$M_t' = \frac{M_t + M_{t-1} + M_{t-2} + ... + M_{t-k+1}}{k}$$

$$a_t = M_t + \left(M_t - M_t'\right) = 2M_t - M_t'$$

$$b_t = \frac{2}{k-1}\left(M_t - M_t'\right)$$

$$\hat{Y}_{t+p} = a_t + b_t p$$

Simple exponential smoothing,

$\hat{Y}_{t+1} = \alpha Y_t + (1-\alpha)\hat{Y}_t$

Holt's linear smoothing

The exponentially smoothed series, or current level estimate,

$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1})$

The trend estimate,

$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$

Forecast p periods into the future,

$\hat{Y}_{t+p} = L_t + pT_t$

Holt-Winter's multiplicative smoothing

The exponentially smoothed series or level estimate,

$$L_t = \alpha \frac{y_t}{S_{t-s}} + (1-\alpha)(L_{t-1} + T_{t-1})$$

The trend estimate,

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

The seasonality estimate,

$$S_t = \gamma \frac{y_t}{L_t} + (1-\gamma)S_{t-s}$$

Forecast p periods into the future,

$$\hat{Y}_{t+p} = (L_t + pT_t)S_{t-s+p}$$

Time additive decomposition,

$$Y_t = S_t + T_t + C_t + I_t$$

Time series multiplicative decomposition,

$$Y_t = S_t * T_t * C_t * I_t$$

AR process: p-order autoregressive model

$$y_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \ldots + \gamma_p y_{t-p} + \varepsilon_t$$

MA process: q-order moving average model

$$y_t = \mu_t + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_q \varepsilon_{t-q}$$

Regression model:

$$t\text{-statistic} = \frac{\hat{\beta} - \beta}{se(\hat{\beta})}$$